Head Office: 216 & 217, 2nd Floor, Grand Plaza, Fraser Road, Dak Bunglow, Patna - 01

**JEE Main 2023 (Memory based)**

**29 January 2023 - Shift 1**

*Answer & Solutions*

## MATHEMATICS

**1.** Consider a function $f(x) = \frac{2x^2+x+1}{x^2+1}$, which of the following options is correct?

A. $f(x)$ is one-one for $x \in (0, 2)$
B. $f(x)$ is many-one for $x \in (0, 2)$
C. $f(x)$ is one-one for $x \in (0, \infty)$
D. $f(x)$ is one-one for $x \in (1, \infty)$

**Answer (A)**

**Solution:**

$f(x) = 2 + \frac{x-1}{x^2+1}$

$f'(x) = -\frac{-x^2+2x+1}{(x^2+1)^2}$

$Q(x) = -x^2 + 2x + 1$ is having $+$ ve sign in interval $(0, 2)$ so function is one-one

**2.** If real part of the product of $z_1$ and $z_2$ is zero, i.e., $Re(z_1 z_2) = 0$ and $Re(z_1 + z_2) = 0$, then $Im(z_1)$ and $Im(z_2)$ is:

*A.* $> 0, > 0$

*B.* $< 0, < 0$

*C.* $= 0, = 0$

*D.* $> 0, < 0$

**Answer (D)**

**Solution:**

Let $z_1 = a_1 + ib_1$ and $z_2 = a_2 + ib_2$

$Re(z_1 z_2) = 0$ $\cdots$ (given)

$\Rightarrow a_1 a_2 = b_1 b_2 \quad \cdots (1)$

$Re(z_1 + z_2) = 0 \Rightarrow a_2 = -a_1 \quad \cdots (2)$

From $(1)$ and $(2)$, $b_1 b_2 = -a_1^2 < 0$

$\Rightarrow Im(z_1)$ and $Im(z_2)$ are of opposite sign.

**3.** Consider $y = f(x)$ passing through $(1, 1)$ satisfying the following differential equation $y(x+1)dx + x^2 dy = 0$, then $y = f(x)$ is given by:

A. $\ln xy = \frac{1}{x} - 1$
B. $\ln xy = \frac{1}{x}$
C. $\ln xy = \frac{1}{x} + 1$
D. $\ln xy = \frac{1}{x^2}$

**Answer (A)**

**Solution:**

$y(x+1)dx + x^2dy = 0$ ⋯ (given)

$y(x+1)dx = -x^2dy$

$\frac{(x+1)}{x^2}dx = -\frac{dy}{y}$

Integrating both sides, we get

$\int \frac{x+1}{x^2}dx = \int -\frac{1}{y}dy \Rightarrow \ln x - \frac{1}{x} = -\ln y + c$

As $y = f(x)$ passes through $(1,\ 1)$

$\Rightarrow \ln 1 - 1 = -\ln 1 + c \Rightarrow c = -1$

$\Rightarrow \ln x - \frac{1}{x} = -\ln y - 1 \Rightarrow \ln x + \ln y = \frac{1}{x} - 1$

$\Rightarrow \ln xy = \frac{1}{x} - 1$

**4.** If $[A]$ is $3 \times 3$ matrix and $A^2 = 3A + aI,\ A^4 = 21A + bI$, then $a + b$ is:

*A.* $-9$

*B.* $-10$

*C.* $9$

*D.* $10$

**Answer (A)**

**Solution:**

$A^4 = A^2 \cdot A^2$

$= (3A + aI)(3A + aI)$

$\Rightarrow A^4 = 9A^2 + 6aA + a^2I = 21A + bI$

Again using $A^2 = 3A + aI$ in LHS

$\Rightarrow 9(3A + aI) + 6aA + a^2I = 21A + bI$

$\Rightarrow (27 + 6a)A + (9a + a^2)I = 21A + bI$

$\therefore 27 + 6a = 21$ & $9a + a^2 = b$

$\therefore a = -1,\ b = -8$

$\Rightarrow a + b = -9$

**5.** Find the area common to following region $x^2 + y^2 \leq 21,\ x \geq 1$ & $y^2 \leq 4x$.

*A.* $8\sqrt{3} - \frac{8}{3} + \frac{21}{2} - \frac{21}{2} sin^{-1}\left(\sqrt{\frac{3}{7}}\right)$

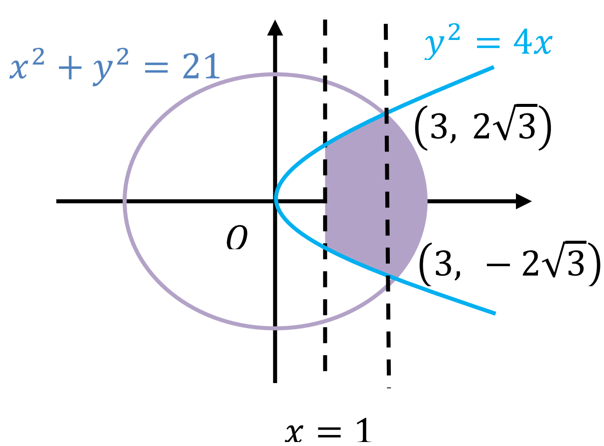
B. $2\sqrt{3} + \frac{21\pi}{2} - \frac{8}{3} - 21\sin^{-1}\left(\sqrt{\frac{3}{7}}\right)$

C. $8\sqrt{3} + \frac{21\pi}{2} - \frac{8}{3}$

D. $8\sqrt{3} + \frac{21\pi}{2} - \frac{8}{3} - 21\sin^{-1}\left(\sqrt{\frac{3}{11}}\right)$

**Answer (B)**

**Solution:**




Area of required region

$= 2\left(\int_1^3 2\sqrt{x}dx + \int_3^{\sqrt{21}} \sqrt{21 - x^2}dx\right)$

$$= 2\left(\left(2 \cdot \frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right)_1^3 + \left[\frac{21 \sin^{-1}\left(\frac{x}{\sqrt{21}}\right) + x\sqrt{21 - x^2}}{2}\right]_3^{\sqrt{21}}\right)$$

$$= 2\left(4\sqrt{3} - \frac{4}{3}\right) + (21 \sin^{-1} 1 + 0) - \left(21 \sin^{-1} \frac{3}{\sqrt{21}} + 3\sqrt{12}\right)$$

$$= 8\sqrt{3} - \frac{8}{3} + \frac{21\pi}{2} - 6\sqrt{3} - 21 \sin^{-1} \sqrt{\frac{3}{7}}$$

$$= 2\sqrt{3} + \frac{21\pi}{2} - \frac{8}{3} - 21 \sin^{-1} \sqrt{\frac{3}{7}}$$

**6.** Domain of $f(x) = \frac{\log_x(x-1)}{\log_{x-1}(x-4)}$ is:

A. $(0,\ 1)$
B. $(4,\ \infty)$
C. $[1,\ 4]$
D. $(4,\ \infty) - \{5\}$

**Answer (D)**

**Solution:**

For domain

$x > 0,\ x - 1 > 0, x \neq 1$

$\&\ x - 1 > 0, x - 1 \neq 1, x - 4 > 0$

$\log_{x-1}(x - 4) \neq 0$

$\Rightarrow x - 4 \neq 1$

$\Rightarrow x \neq 5$

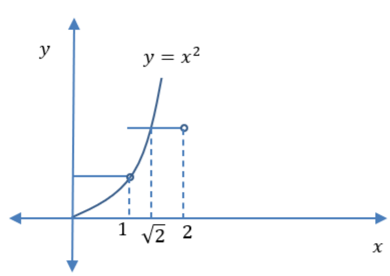
$\therefore x \in (4,\ \infty) - \{5\}$

**7.** Consider $f(x) = \max\{x^2,\ 1 + [x]\}$, where $[x]$ is greatest integer function. Then the value of $\int_0^2 f(x)dx$ is:

A. $\frac{4\sqrt{2}+5}{3}$
B. $\frac{6\sqrt{2}+5}{3}$
C. $\frac{8\sqrt{2}+5}{3}$
D. $\frac{8\sqrt{2}+3}{5}$

**Answer (A)**

**Solution:**




$$f(x) = \begin{cases} 1 + [x],\ 0 \leq x \leq \sqrt{2} \\ x^2,\quad \sqrt{2} < x \leq 2 \end{cases}$$

$$\int_0^2 f(x)dx = \int_0^{\sqrt{2}} (1 + [x])dx + \int_{\sqrt{2}}^2 x^2 dx$$

$= \int_0^1 1dx + \int_1^{\sqrt{2}} 2dx + \frac{x^3}{3}\Big|_{\sqrt{2}}^2$

$= 1 + 2(\sqrt{2} - 1) + \frac{1}{3}(8 - 2\sqrt{2})$

$= \frac{4\sqrt{2}+5}{3}$

**8.** In a football club there are $15$ players, each player has a T-shirt of their own name. Find the number of ways such that at least thirteen players pick the correct T-shirt of their own name.

A. $107$
B. $106$
C. $108$
D. $109$

**Answer (B)**

**Solution:**

At least $13$ players pick correct T-shirt = exactly $13$ players pick correct T-shirt $+14$ players pick correct T-shirt + exactly $15$ players pick correct T-shirt

$= {}^{15}C_{13} \times 1 + 0 + 1$

$= 105 + 0 + 1$

$= 106$

**9.** If $3$ bad and $7$ good apples are mixed, then find probability of finding $4$ good apples if $4$ apples are drawn simultaneously.

A. $\frac{5}{12}$
B. $\frac{1}{6}$
C. $\frac{7}{13}$
D. $\frac{6}{7}$

**Answer (B)**

**Solution:**

$P(E) = \frac{{}^{7}C_4}{{}^{10}C_4} = \frac{7!}{4!3!} \cdot \frac{4!6!}{10!}$

$= \frac{4\cdot5\cdot6}{8\cdot9\cdot10}$

$= \frac{1}{2} \cdot \frac{1}{3} = \frac{1}{6}$

**10.** If $x = 2$ is a root of $x^2 + px + q = 0$ and $f(x) = \begin{cases} \frac{1-\cos(x^2-4px+q^2+8q+16)}{(x-2p)^2}, & x \neq 0 \\ 0, & x = 2p \end{cases}$. Then $\lim_{x\to 2p^+} f(x)$ is:

A. $\frac{1}{2}$
B. $\frac{1}{4}$
C. $0$
D. $-\frac{1}{2}$

**Answer (C)**

**Solution:**

Since $x = 2$ is a root of $x^2 + px + q = 0$

$\Rightarrow 4 + 2p + q = 0$

$\Rightarrow 2p = -4 - q$

$\Rightarrow (q + 4)^2 = 4p^2$

$\lim_{x \to 2p^+} f(x) = \lim_{x \to 2p^+} \frac{1-\cos(x^2-4px+q^2+8q+16)}{(x-2p)^2}$

$= \lim_{x \to 2p^+} \frac{1-\cos(x^2-4px+(q+4)^2)}{(x-2p)^2}$

$= \lim_{x \to 2p^+} \frac{1-\cos(x^2-4px+4p^2)}{(x-2p)^2}$

$= \lim_{x \to 2p^+} \frac{1-\cos(x-2p)^2}{(x-2p)^2}$

$= \lim_{x \to 2p^+} \frac{2\sin^2\frac{(x-2p)^2}{2}}{(x-2p)^2} \times \frac{(x-2p)^2 \times 4}{4 \times (x-2p)^2} \left(\because\ 1 - \cos\theta = 2\sin^2\frac{\theta}{2}\right)$

$= \lim_{x \to 2p^+} 2\left(\frac{\sin\frac{(x-2p)^2}{2}}{\frac{(x-2p)^2}{2}}\right) \times \frac{(x-2p)^2}{4}$

$= 0$

**11.** If Incident ray $y = \frac{x}{\sqrt{3}}$ is incident on a reflecting surface $x + y = 1$. Then point of intersection of reflecting ray with $x$-axis is:

*A.* $\left(1 - \frac{1}{\sqrt{3}}, 0\right)$

*B.* $\left(1 + \frac{1}{\sqrt{3}}, 0\right)$

*C.* $\left(\frac{1}{\sqrt{3}}, 0\right)$

*D.* $\left(\frac{2}{\sqrt{3}}, 0\right)$

**Answer (A)**

**Solution:**

Solving,

$x + y = 1$ &

$y = \frac{x}{\sqrt{3}}$

$x + \frac{x}{\sqrt{3}} = 1$

$x = \frac{\sqrt{3}}{\sqrt{3}+1}$

$y = \frac{1}{\sqrt{3}+1}$

$\Rightarrow A = \left(\frac{\sqrt{3}}{\sqrt{3}+1}, \frac{1}{\sqrt{3}+1}\right)$

For line $AP$, $y - \frac{1}{\sqrt{3}+1} = \sqrt{3}\left(x - \frac{\sqrt{3}}{\sqrt{3}+1}\right)$

At $P$, $-\frac{1}{\sqrt{3}+1} = \sqrt{3}\left(h - \frac{\sqrt{3}}{\sqrt{3}+1}\right)$

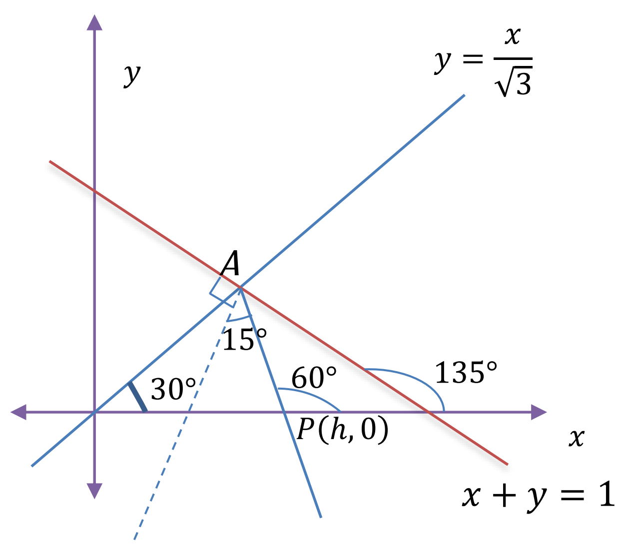
$\Rightarrow h = -\frac{1}{\sqrt{3}(\sqrt{3}+1)} + \frac{\sqrt{3}}{\sqrt{3}+1}$

$= \frac{1}{\sqrt{3}+1}\left(-\frac{1}{\sqrt{3}} + \sqrt{3}\right)$

$= \frac{2}{\sqrt{3}(\sqrt{3}+1)}$

$= \frac{\sqrt{3}-1}{\sqrt{3}}$

$\Rightarrow h = 1 - \frac{1}{\sqrt{3}}$

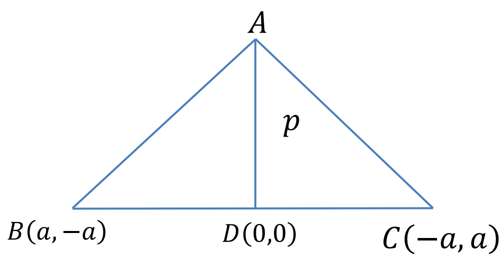


**12.** In an equilateral triangle $ABC$, point $A$ lies on the line $y - 2x = 2$ and points $B$ and $C$ are lying on the line $y + x = 0$. If Points $B$ and $C$ are symmetric with respect to origin then area of the $\Delta ABC$ (in sq. units) is :

A. $4\sqrt{3}$
B. $8$
C. $\frac{8}{\sqrt{3}}$
D. $8\sqrt{3}$

**Answer (A)**

**Solution:**




Let coordinates of points B & C be $(a, -a)$& $(-a, a)$ respectively.

$A$ lies on perpendicular bisector of $BC$

$i.e., \Rightarrow y = x$

$A$ is point of intersection of $y = x$ and $y - 2x = 2$

$A = (-2, -2)$

$p = AD = 2\sqrt{2}$

Area of $\Delta ABC = \frac{1}{2} . \frac{2p}{\sqrt{3}} . p$

$= \frac{p^2}{\sqrt{3}} = \frac{8}{\sqrt{3}}$(sq. units)

**13.** It is given that $\left((p \wedge q) \vee r\right) \vee (p \wedge r) \rightarrow (\sim q) \vee r$ is fallacy. Then truth values of $p$, $q$ and $r$ are given by:

A. $p$: true, $q$: true, $r$: false
B. $p$: false, $q$: false, $r$: false
C. $p$: true, $q$: true, $r$: true
D. None of these

**Answer (A)**

**Solution:**

$s \rightarrow t$ is always false if $s$ is true and $t$ is false.

$\therefore (\sim q) \vee r$ is false $\Rightarrow \sim q$ is false and $r$ is false

$\Rightarrow q$ is true and $r$ is false.

Also, if $p$ is true, then $\left((p \wedge q) \vee r\right) \vee (p \wedge r)$ is true.

$\therefore$ option $A$ is correct.

**14.** Let $A$ & $B$ be area of regions for $x \in [0, 1]$ given by

$A: 2x \le y \le \sqrt{4(x-1)^2}$ with $y$-axis

$B: y = \min\left\{2x, \sqrt{4(x-1)^2}\right\}$ with $x$-axis, then $\frac{A}{B}$ equals:

A. $1$
B. $2$
C. $3$
D. $4$

**Answer (A)**

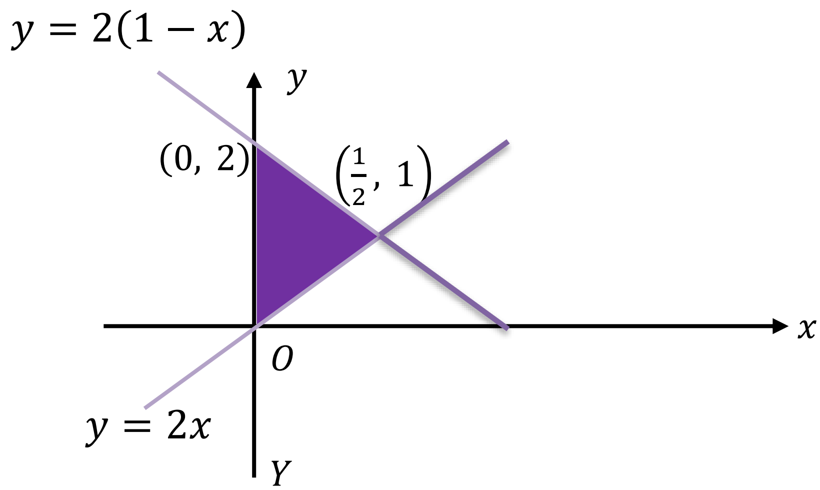
**Solution:**

$A: 2x \le y \le \sqrt{4(x-1)^2}$ with $y$-axis

$y \le \sqrt{4(x-1)^2} \Rightarrow y \le 2|x-1|$

$\Rightarrow y \le 2(1-x) \because x \in [0, 1]$

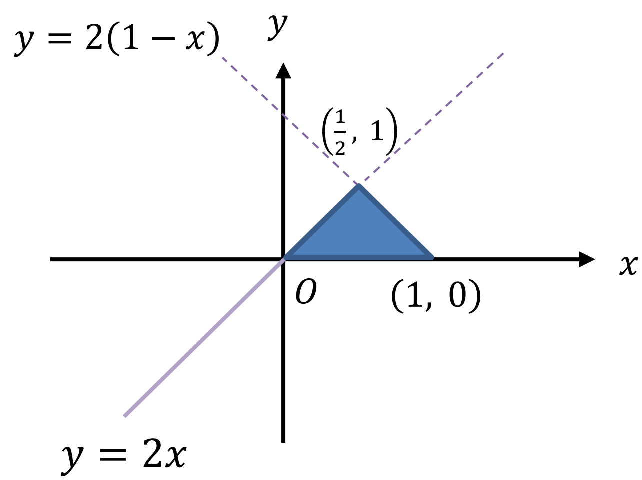
From the figure $A$ is equal to area of highlighted triangle $A = \frac{1}{2} \times 2 \times \frac{1}{2} = \frac{1}{2}$




$B: y = \min\left\{2x, \sqrt{4(x-1)^2}\right\}$ with $x$-axis:

From this figure we get $B$ as:

$B = \frac{1}{2} \times 1 \times 1 = \frac{1}{2}$




$\therefore \frac{A}{B} = 1$

**15.** A function $f(x)$ is such that $f(x+y) = f(x) + f(y) - 1 \; \forall x, y \in \mathbb{R}$, Also $f'(0) = 2$, then $|f(-2)|$ is:

**Answer (3)**

**Solution:**

$f(x+y) = f(x) + f(y) - 1$

Put $x = y = 0$

$\Rightarrow f(0) = f(0) + f(0) - 1$

$\Rightarrow f(0) = 1$

Now $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$\Rightarrow f'(x) = \lim_{h \to 0} \frac{f(x) + f(h) - 1 - f(x)}{h}$ $\quad [\because f(x+y) = f(x) + f(y) - 1]$

$\Rightarrow f'(x) = \lim_{h \to 0} \frac{f(h) - 1}{h}$

$\Rightarrow f'(x) = f'(0) = 2 \quad \cdots (\because f'(0) = 2)$

Integrating both sides

$\Rightarrow f(x) = 2x + c$

$\because f(0) = 1,$

$\Rightarrow c = 1$
$\therefore f(x) = 2x + 1$
$f(-2) = (2 \times -2) + 1 = -3$
$|f(-2)| = 3$

**16.** If $a_1,\ a_2, \ldots$ are positive numbers in GP such that $a_5 + a_7 = 12$ and $a_4 . a_6 = 9$ then $a_7 + a_9$ equals ______.

**Answer (36)**

**Solution:**

Let first term of GP be $a$ with common ratio $r$
$\Rightarrow a, r > 0$
$a_5 + a_7 = 12 \cdots (1)$
$a_4 . a_6 = 9$
$ar^3 \cdot ar^5 = 9$
$\Rightarrow a^2 r^8 = 9$
$\Rightarrow ar^4 = 3 \Rightarrow a_5 = 3$
Substitute $a_5 = 3$ in eq. $(1)$
$3 + a_7 = 12$
$\Rightarrow a_7 = 9$
$\Rightarrow ar^6 = 9$
By taking the ratio of $\frac{a_7}{a_5}$, we get
$\frac{a_7}{a_5} = \frac{ar^6}{ar^4} = \frac{9}{3}$
$\Rightarrow r^2 = 3$
$\Rightarrow r = \sqrt{3}$
$\Rightarrow a = \frac{1}{3}$
$\therefore a_9 = ar^8$
$\Rightarrow a_9 = \frac{1}{3} \times \left(\sqrt{3}\right)^8 = 27$
$\therefore a_7 + a_9 = 9 + 27 = 36$

**17.** If $f(x+y) = f(x) + f(y),\ f(1) = \frac{1}{5}$ and $\sum_{n=1}^{N} \frac{f(n)}{n(n+1)(n+2)} = \frac{1}{12}$, then the value of $N$ is ______.

**Answer (10)**

**Solution:**

$f(x+y) = f(x) + f(y) \qquad \cdots$ (given)
$\Rightarrow f(x) = kx$
$f(1) = \frac{1}{5} \Rightarrow k = \frac{1}{5}$
$\therefore f(x) = \frac{1}{5}x$
$\sum_{n=1}^{N} \frac{f(n)}{n(n+1)(n+2)} = \frac{1}{12} \quad \cdots$ (given)
$\Rightarrow \sum_{n=1}^{N} \frac{\frac{1}{5}n}{n(n+1)(n+2)} = \frac{1}{12}$
$\Rightarrow \frac{1}{5} \sum_{n=1}^{N} \left(\frac{1}{n+1} - \frac{1}{n+2}\right) = \frac{1}{12}$
$\Rightarrow \frac{1}{5}\left(\frac{1}{2} - \frac{1}{N+2}\right) = \frac{1}{12}$
$\Rightarrow \frac{N}{10(N+2)} = \frac{1}{12}$
$\Rightarrow 12N = 10N + 20$
$\Rightarrow 2N = 20$
$\Rightarrow N = 10$

**18.** Let $S = \{1,\ 2,\ 3,\ 5,\ 7\}$. The rank of $35773$, if all $5$ digit numbers formed by the set $S$ are arranged in a dictionary in ascending order & repetition of digits is allowed is __________.

**Answer (1748)**

**Solution:**

All five digit numbers starting from $1$ and $2$ will come first

i.e., $1 - \ - \ - - \rightarrow 5^4$

$2 - \ - \ - \ - \rightarrow 5^4$

If first digit is $3$ (number of numbers that comes before $35773$)

$3\ 1\ - \ - - \rightarrow 5^3$

$3\ 2 - - - \rightarrow 5^3$

$3\ 3 - - - \rightarrow 5^3$

$3\ 5\ 1 - - \rightarrow 5^2$

$3\ 5\ 2 - - \rightarrow 5^2$

$3\ 5\ 3 - - \rightarrow 5^2$

$3\ 5\ 5 - - \rightarrow 5^2$

$3\ 5\ 7\ 1 - \rightarrow 5$

$3\ 5\ 7\ 2 - \rightarrow 5$

$3\ 5\ 7\ 3 - \rightarrow 5$

$3\ 5\ 7\ 5\ - \rightarrow 5$

$3\ 5\ 7\ 7\ 1 \rightarrow 1$

$3\ 5\ 7\ 7\ 2 \rightarrow 1$

$3\ 5\ 7\ 7\ 3 \rightarrow 1$

$\therefore$ rank $= 2(5^4) + 3(5^3) + 4(5^2) + 4(5) + 3$

$= 1250 + 375 + 100 + 20 + 3$

rank $= 1748$

**19.** If the ratio of coefficients of $3$ consecutive terms in expansion of $(1+2x)^n$ is $10:35:84$. Then $n$ is equal to __________.

**Answer (10)**

**Solution:**

$\frac{{}^nC_r 2^r}{{}^nC_{r+1} 2^{r+1}} = \frac{2}{7}$

$\Rightarrow \frac{r+1}{n-r} \cdot \frac{1}{2} = \frac{2}{7}$

$\Rightarrow n - r = \frac{7}{4}(r+1)$ ... $(1)$

$\frac{{}^nC_{r+1} 2^{r+1}}{{}^nC_{r+2} 2^{r+2}} = \frac{5}{12}$

$\Rightarrow \frac{r+2}{n-r-1} \cdot \frac{1}{2} = \frac{5}{12}$

$\Rightarrow n - r - 1 = \frac{6}{5}(r+2) \quad \cdots (2)$

Solving eq.$(1)$ and eq.$(2)$

$r = 3$ & $n = 10$

**20.** Consider $3$ coplanar vectors $\vec{a} = 3\hat{i} - 4\hat{j} + \lambda\hat{k}$, $\vec{b} = 4\hat{i} + 3\hat{j} - \hat{k}$ and $\vec{c} = \hat{i} + 3\hat{j} - 4\hat{k}$. Then $9\lambda$ is ________.

**Answer (87)**

**Solution:**

For coplanar vectors

$$\begin{vmatrix} 3 & -4 & \lambda \\ 4 & 3 & -1 \\ 1 & 3 & -4 \end{vmatrix} = 0$$

$\Rightarrow 3(-12+3) + 4(-16+1) + \lambda(12-3) = 0$

$\Rightarrow -27 - 60 + 9\lambda = 0$

$\Rightarrow 9\lambda = 87$